AP Calculus AB

L'Hopital's Rule

1) $\lim_{x\to 2} \frac{x^3-8}{x^2-4} \to \frac{0}{0}$

Using L'Hopital's Rule

$\lim_{x\to 2} \frac{3x^2}{2x} = 3$

2) $\lim_{x\to 0} \frac{\sin 5x}{x} \to \frac{0}{0}$

Using L'Hopital's Rule

$\lim_{x\to 0} \frac{5\cdot\cos 5x}{1} = 5$

3) $\lim_{x\to 2} \frac{\sqrt{2+x}-2}{x-2} \to \frac{0}{0}$

Using L'Hopital's Rule

$\lim_{x\to 2} \frac{\frac{1}{2}(x+2)^{-1/2}}{1} = \frac{1}{4}$

4) $\lim_{x\to 1} \frac{\sqrt[3]{x}-1}{x-1} \to \frac{0}{0}$

Using L'Hopital's Rule

$\lim_{x\to 1} \frac{\frac{1}{3}x^{-2/3}}{1} = \frac{1}{3}$

5) $\lim_{x\to 2} \frac{x^2-4x+4}{x^3-12x+16} \Rightarrow \frac{0}{0}$

Using L'Hopital's Rule

$\lim_{x\to 2} \frac{2x-4}{3x^2-12} \to \frac{0}{0}$

$\lim_{x\to 2} \frac{2}{6x} = \frac{1}{6}$

6) $\lim_{x\to \frac{\pi}{2}} \frac{1-\sin x}{1+\cos 2x} \to \frac{0}{0}$

Using L'Hopital's Rule

$\lim_{x\to \frac{\pi}{2}} \frac{-\cos x}{-2\sin 2x} = \frac{0}{0}$

$\lim_{x\to \frac{\pi}{2}} \frac{\sin x}{-4\cos 2x} = \frac{1}{4}$

7) $\lim_{x\to 1} \frac{x^3-1}{4x^3-x-3} \to \frac{0}{0}$

Using L'Hopital's Rule

$\lim_{x\to 1} \frac{3x^2}{12x^2-1} = \frac{3}{11}$

8) $\lim_{x\to 3} \frac{x-4}{x-2} = -1$

9) $f(x) = e^{\sqrt{x}} = e^{\left|x^{1/2}\right|}$

$f'(x) = e^{x^{1/2}} \cdot \left(\frac{1}{2}x^{-1/2}\right)$

$= \frac{e^{\sqrt{x}}}{2\sqrt{x}}$

10) $y = e^{\left|4\ln x\right|}$

LOG PROPERTY

$y' = e^{4\ln x} \cdot 4\left(\frac{1}{x}\right) = \frac{4e^{4\ln x}}{x} = \frac{4\left[e^{\ln x^4}\right]}{x}$

$= \frac{4x^4}{x} = \boxed{4x^3}$

11) $y = \ln\left|\sin x\right|$

$y' = \frac{\cos x}{\sin x} = \cot x$

12) $y = \sqrt{4x^2+4x} = (4x^2+4x)^{1/2}$

$y' = \frac{1}{2}(4x^2+4x)^{-1/2} \cdot (8x+4)$

$= \frac{4x+2}{\sqrt{4x^2+4x}} = \frac{4x+2}{\sqrt{4(x^2+x)}} = \frac{4x+2}{2\sqrt{x^2+1}}$

$= \frac{2x+1}{\sqrt{x^2+x}}$

13) $g(x) = \cos^3 5x = [\cos 5x]^3$

$g'(x) = 3[\cos 5x]^2 \cdot d[\cos 5x]$

$= 3[\cos 5x]^2 \cdot [-\sin 5x \cdot d(5x)]$

$= 3(\cos 5x)^2 \cdot [-5 \sin 5x]$

$= -15 \cos^2 5x \sin 5x$

14) $y = xe^{2x}$

$y' = x \cdot e^{2x} \cdot 2 + e^{2x}$

$= 2x e^{2x} + e^{2x}$

$y' = e^{2x}(2x+1)$

15) $\sqrt[3]{25}$

$f(x) = \sqrt[3]{x}$ $a = 27$

point: $(27, 3)$

slope: $f'(x) = \frac{1}{3}x^{-2/3}$

$f'(27) = \frac{1}{3} \cdot \left(\frac{1}{9}\right) = \frac{1}{27}$

$y - 3 = \frac{1}{27}(x - 27)$

$L(x) = 3 + \frac{1}{27}(x-27)$

$L(25) = 3 + \frac{1}{27}(-2)$

$\boxed{f(25) \approx 3 - \frac{2}{27}}$

16) $f(x) = e^x$; $a = 0$

$f(0) = 1$ $f'(x) = e^x$

$f'(0) = 1$

$y - 1 = 1(x - 0)$

$L(x) = 1 + 1(x-0)$

$L(0.1) = 1 + 1(0.1)$

$\boxed{f(0.1) \approx 1.1}$